Particle acceleration in an active medium

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It is shown that a bunch of electrons which move in an active medium can be accelerated. The acceleration is proportional to the population inversion and the number of electrons in the bunch. As reference we compare the acceleration force with the deceleration experienced by the same bunch as it traverses a dielectric and metallic medium. [S1063-651X(96)08005-1]

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I. INTRODUCTION

When an electron moves along a vacuum channel in a dielectric material it may cause radiation to be emitted provided that its velocity is greater than the phase velocity of an electromagnetic plane wave in the medium. This is the so-called Čerenkov radiation. What a remote observer measures as electromagnetic energy comes at the expense of the particle's kinetic energy or, in other words, the particle is decelerated. For a better understanding of the deceleration force, one has to examine the field distribution in the vicinity of the particle. Ignoring for a moment the presence of the dielectric, a point charge generates in its rest frame of reference an electrostatic field which transforms in the laboratory frame into an infinite spectrum of evanescent waves. As these waves hit the discontinuity between the vacuum channel and the dielectric, a so-called secondary field is generated. This is the reaction of the medium to the presence of the charged particle. It is the action of this secondary field which decelerates the electron. In this study it will be shown that if, instead of a passive dielectric medium, an active medium is used, the action of this secondary field may cause the particle to accelerate. Thus energy stored in the medium can be transferred to the moving electron.

An additional way to examine the proposed acceleration scheme is to consider the microscopic processes. In particular, let us consider an ensemble of atoms of which each one is modeled by a two-level system. The number of atoms in which the electron is in the upper level is larger than the number of these in the lower level, i.e., the population is inverted. As indicated above, attached to a moving charge there is a broadband spectrum of evanescent waves, including the resonance frequency corresponding to the two-level system. These waves can be conceived as a spectrum of virtual photons continuously emitted and absorbed by the electron. When a virtual photon corresponding to the resonance frequency impinges upon an excited atom, its effect is identical to that of a regular photon: it stimulates the atom and two identical (phase correlated) photons are emitted. One photon is identical with the initial and the second is a real photon. Since the two are phase correlated, the real photon can be absorbed by the moving electron, causing the latter's acceleration. The inverse process is also possible: if the virtual photon encounters an atom in the ground state and excites it, then the moving electron loses energy, i.e., it is being slowed down. We may expect net acceleration only if the number of atoms in the excited state is larger than these in the lower state, i.e., the population is inverted [1]. From the description above, the acceleration force is a result of stimulated radiation, therefore we call this scheme PASER, which stands for particle acceleration by stimulated emission of radiation.

Many of the advanced acceleration concepts have lasers as their central component. The main schemes follow. (i) Beat-wave accelerator (BWA) in which two slightly different laser beams illuminate a plasma whose frequency equals the difference of the two laser frequencies. The injected particles are accelerated by the resonant space charge wave which develops in the system [2]. (ii) Wake-field acceleration (WFA), suggested by Sprangle *et al.* [3], is based on a large wake field left after a very intense but short laser impulse which propagates in plasma. (iii) Inverse Čerenkov [4] relies on the Čerenkov effect to accelerate particles. These are injected in a slow-wave environment (gas) and illuminated at the Čerenkov angle by an adequate laser beam. A concept which is similar, but is based on a fast-wave interaction, is the (iv) inverse free electron laser (IFEL) [5]. In all the above schemes the radiation is generated in the laser cavity, it is guided by optics to the interaction region, and then it is utilized to accelerate the electrons in one way or another. In fact, in the last two cases we also use the inverse of a radiation process. The question which we asked originally was whether one can directly use the ''inverse laser'' process for particle acceleration. According to the detailed calculation presented here, the answer to this question is affirmative.

In the past, there were two schemes in which an active medium was suggested in order to support the acceleration process [6,7]. In both cases the active medium facilitates the generation of solitons, extending the interaction region in this manner. To illustrate the role of the active medium let us consider the wake-field acceleration scheme. It has been shown that high gradients may develop in the plasma; however, the interaction with electrons alters the propagation characteristics of the medium and the overlap of the electrons with the laser beam is limited. Fisher and Tajima [7] have shown that an active medium can be used in order to preserve the radiation pulse shape, energy, and velocity. The scheme relies partially on the self-induced transparency theory which was first developed by McCall and Hahn [8] in 1969. Fisher and Tajima envision their system to use the outer shell electrons to form the plasma and the inner shell electron resonant transition as the constituent of the active

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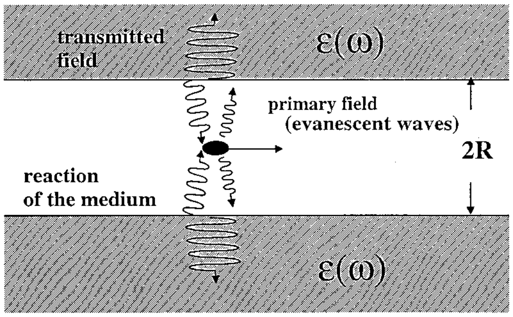

FIG. 1. The schematic of the system used to examine the gradient which acts on a bunch of electrons as they move in a vacuum channel bored in a dielectric medium. The latter can be complex and frequency dependent.

medium. As previously indicated, in this study it will be shown that the active medium can be utilized not only to preserve the laser pulse shape but also to directly accelerate the electrons; in fact, no plasma is required in the PASER scheme.

Another acceleration scheme which is under investigation and has some resemblance to the scheme proposed here is the dielectric wake-field accelerator (DWFA), demonstrated in 1988 by Gai *et al.* [9]. It relies on the injection of an intense electron pulse, of low voltage high current, into a dielectrically loaded waveguide. The wake (Čerenkov radiation) left by the driving pulse accelerates another bunch which trails and it is characterized by high voltage and low current. The approach was originally considered by Voss and Weiland [10] except that the structure was periodic and the driving beam was annular. In the PASER scheme the energy is initially stored in the medium and it is stimulated by the accelerated bunch itself—thus eliminating the need for synchronization of the two bunches.

This paper is organized as follows: in the next section we present a general formulation of the force which acts on an electron as it moves in a vacuum channel bored in a dielectric material. After this general formulation we present two simple cases of a passive dielectric medium and lossy material. This is followed by analysis of the motion in an active medium.

II. GENERAL FORMULATION

The radiation emitted when a particle moves with a velocity which exceeds the phase velocity of the electromagnetic wave in the medium comes at the expense of its kinetic energy. In order to understand the source of this force we have to realize that the field of a moving particle consists of a superposition of evanescent waves. As the particle moves in a vacuum channel of radius R surrounded by a dielectric medium ε_r, the evanescent waves hit the discontinuity at $r=R$ and they are partially reflected and partially transmitted—see Fig. 1. It is the reflected waves which act back on the electron, decelerating it. In this section we shall examine this process for a scalar dielectric coefficient which in principle can be complex and frequency dependent.

Consider a point charge (q) moving at a constant velocity V whose current density is described by

$$J_z(r,z,t)=-qV\,\frac{1}{2\pi r}\,\delta(r)\,\delta(z-Vt), \tag{1}$$

and its time Fourier transform by

$$J_z(r,z,\omega)=-\frac{q}{(2\pi)^2 r}\,\delta(r)e^{-j(\omega/V)z}. \tag{2}$$

The magnetic vector potential, in the frequency domain, determined by this current density is given by

$$\begin{aligned} A_z&(r<R,z,\omega)\\ &=2\pi\mu_0\int_{-\infty}^{\infty}dz'\int_0^R dr'\,r'\,G(r,z|r',z')J_z(r',z',\omega)\\ &\quad+\int_{-\infty}^{\infty}dk\;\varrho(k)e^{-jkz}I_0(\Gamma r) \end{aligned} \tag{3}$$

and

$$A_z(r>R,z,\omega)=\int_{-\infty}^{\infty}dk\;\tau(k)e^{-jkz}K_0(\Lambda r), \tag{4}$$

where $\Gamma^2=k^2-(\omega/c)^2$, $\Lambda^2=k^2-\varepsilon_r(\omega/c)^2$, and $G(r',z'|r,z)$ is the boundless Green's function, i.e.,

$$G(r'z'|r,z)=\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}dk\;e^{-jk(z-z')}\begin{cases} I_0(\Gamma r)K_0(\Gamma r') & \text{for } 0<r<r'<\infty\\ K_0(\Gamma r)I_0(\Gamma r') & \text{for } 0<r'<r<\infty \end{cases}. \tag{5}$$

The amplitudes ϱ and τ represent the reflected and transmitted waves correspondingly. In order to determine these amplitudes we have to impose the boundary conditions at $r=R$. For this purpose it is convenient to write the solution of the magnetic vector potential off axis as

$$\begin{aligned} A_z&(0<r<R,z,\omega)\\ &=\int_{-\infty}^{\infty}dk\;e^{-jkz}[\varrho(k)I_0(\Gamma r)+\alpha(k)K_0(\Gamma r)], \end{aligned} \tag{6}$$

where

$$\alpha(k)=-\frac{q\mu_0}{(2\pi)^2}\,\delta\!\left(k-\frac{\omega}{V}\right). \tag{7}$$

From the continuity of the longitudinal electric field (E_z) we conclude that

$$\frac{c^2}{j\omega}\left[\frac{\omega^2}{c^2}-k^2\right][\varrho(k)I_0(\Gamma R)+\alpha(k)K_0(\Gamma R)]$$
$$=\frac{c^2}{j\omega\varepsilon_r}\left[\varepsilon_r\frac{\omega^2}{c^2}-k^2\right]\tau(k)K_0(\Lambda R). \tag{8}$$

In a similar way the continuity of the azimuthal magnetic field implies

$$\Gamma[\varrho(k)I_1(\Gamma R)-\alpha(k)K_1(\Gamma R)]=-\Lambda\tau(k)K_1(\Gamma R). \tag{9}$$

At this stage we introduce the (normalized) impedances ratio

$$\zeta\equiv\frac{1}{\varepsilon_r}\frac{\Lambda}{\Gamma}\frac{K_0(\Lambda R)}{K_1(\Lambda R)}, \tag{10}$$

by whose means the amplitudes of the reflected waves are

$$\varrho=\alpha\,\frac{\zeta K_1(\Gamma R)-K_0(\Gamma R)}{\zeta I_1(\Gamma R)+I_0(\Gamma R)}. \tag{11}$$

The only nonzero field on axis is the longitudinal electric field and only the waves ''reflected'' from the radial discontinuity contribute to the force which acts on the particle, therefore

$$E_z(r=0,z=Vt,t)=\int_{-\infty}^{\infty}d\omega\,dk\,\frac{c^2}{j\omega}\left[\frac{\omega^2}{c^2}-k^2\right]\times\varrho(\omega,k)e^{j(\omega-kV)t}. \tag{12}$$

Substituting the explicit expression for ϱ, using the integral over the Dirac δ function, and defining $x=\omega R/c\beta\gamma$, we obtain

$$E_z(r=0,z=Vt,t)=\frac{-jq}{(2\pi)^2\varepsilon_0R^2}\int_{-\infty}^{\infty}dx\;x\,\frac{\zeta(x)K_1(|x|)-K_0(|x|)}{\zeta(x)I_1(|x|)+I_0(|x|)}. \tag{13}$$

At this point it is convenient to define the normalized field which acts on the particle as

$$\mathscr{E}\equiv E_z(r=0,z=Vt,t)\left(\frac{q}{4\pi\varepsilon_0R^2}\right)^{-1}=\frac{2}{\pi}\int_0^{\infty}dx\;x\,\mathrm{Re}\left[\frac{1}{j}\,\frac{\zeta(x)K_1(|x|)-K_0(|x|)}{\zeta(x)I_1(|x|)+I_0(|x|)}\right]. \tag{14}$$

Clearly from this representation we observe that for a nonzero force to act on the particle the impedance ratio ζ has to be complex since the argument of the modified Bessel functions is real.

We can make one step further and simplify this expression by defining

$$\zeta(x)\equiv|\zeta(x)|e^{j\psi(x)}, \tag{15}$$

and using $K_0(x)I_1(x)+K_1(x)I_0(x)=1/x$, we obtain

$$\mathscr{E}=\frac{2}{\pi}\int_0^{\infty}dx\,\frac{|\zeta(x)|\sin\psi(x)}{I_0^2(x)+|\zeta(x)|^2I_1^2(x)+2|\zeta(x)|I_0(x)I_1(x)\cos\psi(x)}=\frac{2}{\pi}\int_0^{\infty}dx\,\frac{\mathrm{Im}[\zeta(x)]}{\{I_0(x)+\mathrm{Re}[\zeta(x)]I_1(x)\}^2+\{\mathrm{Im}[\zeta(x)]I_1(x)\}^2}. \tag{16}$$

We shall use this expression to evaluate the force which acts on a moving charge for three cases; in the next section for the Čerenkov radiation case, followed by the lossy (Ohmic) medium, and then by the active medium.

III. ČERENKOV FORCE

In order to evaluate the integral in (16) for a dielectric medium and a particle whose velocity βc is larger than $c/\sqrt{\varepsilon_r}$, we go back to the expression for the normalized impedance in (10) which now reads

$$\zeta(x)=j\,\frac{\gamma}{\varepsilon_r}\sqrt{\varepsilon_r\beta^2-1}\,\frac{K_0(jx\gamma\sqrt{\varepsilon_r\beta^2-1})}{K_1(jx\gamma\sqrt{\varepsilon_r\beta^2-1})}. \tag{17}$$

With this explicit expression the integral for the normalized gradient can be calculated numerically and the result is illustrated in the two frames of Fig. 2. Analytic expressions can be found if we assume that the main contribution occurs for large arguments of the Bessel function, implying that primarily short (compare to the radius) wavelengths contribute, i.e., $(\omega R/c)\sqrt{\varepsilon_r-\beta^{-2}}\gg1$, hence

$$\zeta(x)\simeq j\,\frac{\gamma}{\varepsilon_r}\sqrt{\varepsilon_r\beta^2-1}. \tag{18}$$

Since subject to this assumption $\psi=\pi/2$ and $|\zeta|$ is constant we can evaluate $\mathscr{E}$,

$$\mathscr{E}=\frac{2}{\pi}\int_0^{\infty}dx\,\frac{|\zeta(x)|}{I_0^2(x)+|\zeta(x)|^2I_1^2(x)}, \tag{19}$$

for two regimes: first when $|\zeta|\gg1$, i.e., $\gamma\gg1$, the contribution to the integral is primarily from small values of x, thus

$$\mathscr{E}\simeq\frac{2}{\pi}\int_0^{\infty}dx\,\frac{|\zeta|}{I+|\zeta|^2x^2/4}\simeq\frac{4}{\pi}\int_0^{\infty}du\,\frac{1}{1+u^2}\simeq2. \tag{20}$$

The same integral at the other extreme ($|\zeta|\ll1$) reads

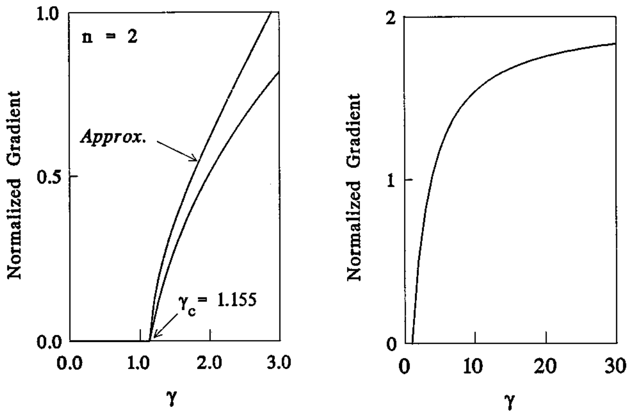

FIG. 2. The normalized gradient which acts on the bunch as it moves in a lossless dielectric medium. The left frame illustrates the force at low energies and the right frame at high energies.

$$\mathscr{E} \simeq \frac{2}{\pi} |\zeta| \int_0^\infty dx \, \frac{1}{I_0^2(x)} \simeq 0.871|\zeta|, \tag{21}$$

and overall

$$\mathscr{E} \simeq \begin{cases} 0 & \text{for } \beta < 1/\sqrt{\varepsilon_r} \\ 0.871\gamma\sqrt{\varepsilon_r\beta^2 - 1}/\varepsilon_r & \text{for } \gamma \ll \varepsilon_r/\sqrt{\varepsilon_r\beta^2 - 1} \\ 2 & \text{for } \gamma \gg \varepsilon_r/\sqrt{\varepsilon_r\beta^2 - 1}. \end{cases} \tag{22}$$

These two expressions are in good agreement with the numerical result illustrated in Fig. 2. It is interesting to note that for ultrarelativistic electrons the decelerating Čerenkov force reaches an asymptotic value which is independent of γ and the dielectric coefficient ε_r; it is given by $E = q/2\pi\varepsilon_0 R^2$. For what follows it is important to observe that the value of the normalized impedance (ζ) determines the force that the point charge experiences in the intermediary regime.

IV. OHMIC FORCE

If in the Čerenkov case the electron has to exceed a certain velocity in order to generate radiation and therefore to experience a decelerating force, then in the case of a lossy medium the moving electron experiences a decelerating force starting from vanishingly small velocity. This is because it induces currents in the surrounding walls and as a result power is dissipated—which is equivalent to the emitted power in the Čerenkov case. The source of this power is the **J**·**E** term which infers the existence of a decelerating force acting on the electron. In order to evaluate this force we use the formulation developed in Sec. II only that in this case the dielectric coefficient is complex and it is given by

$$\varepsilon_r = 1 - j\,\frac{\sigma}{\varepsilon_0\omega}, \tag{23}$$

where σ is the (finite) conductivity of the surrounding medium. It is convenient to use the same notation as above, therefore the normalized impedance ζ from (18) is replaced by

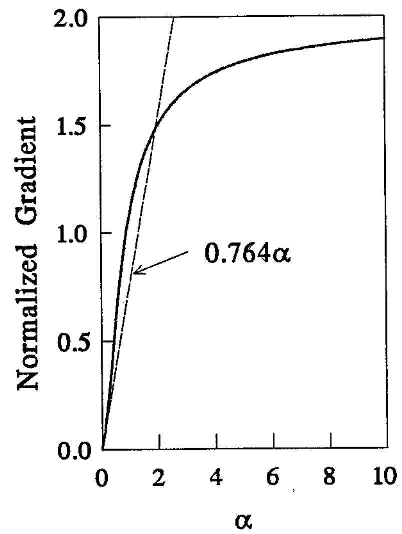

FIG. 3. The normalized gradient experienced by a bunch of electrons as it moves in a vacuum channel bored in a lossy medium. The parameter α is defined by $\alpha = \sqrt{(\gamma\beta)^3/\sigma\eta_0 R}$.

$$\zeta \simeq \frac{1}{1 - j\sqrt{\bar{\sigma}/x}} \sqrt{1 + j(\gamma\beta)^2 \frac{\bar{\sigma}}{x}}. \tag{24}$$

In this expression $\bar{\sigma} \equiv \sigma\eta_0 R/\gamma\beta$ which for typical metals and $R \sim 1$ cm is on the order of $10^7/\gamma\beta$, thus for any practical purpose $\bar{\sigma} \gg 1$. Hence

$$\zeta \simeq \gamma\beta e^{j3\pi/4} \sqrt{\frac{x}{\bar{\sigma}}}. \tag{25}$$

Note that the phase of the normalized impedance is $\psi = 3\pi/4$. Substituting this expression in (16) we obtain

$$\mathscr{E} = \frac{2}{\pi} \int_0^\infty dx \, \frac{\alpha\sqrt{x}}{[I_0(x) - \alpha\sqrt{x} I_1(x)]^2 + [\alpha\sqrt{x} I_1(x)]^2}, \tag{26}$$

where $\alpha \equiv \sqrt{(\gamma\beta)^3/\sigma\eta_0 R}$. This integral was evaluated numerically and the result of $\mathscr{E}$ as a function of α is plotted in Fig. 3. The integral can be evaluated analytically for two extreme regimes: in the first case the (normalized) momentum of the particle is assumed to be much smaller than the normalized conductivity term, i.e., $(\gamma\beta)^3 \ll \sigma\eta_0 R$ thus $\alpha \ll 1$, in which case

$$\mathscr{E} \simeq \alpha \left[\frac{2}{\pi} \frac{\sqrt{2}}{2} \int_0^\infty dx \, \frac{\sqrt{x}}{I_0^2(x)} \right] \simeq 0.764\alpha. \tag{27}$$

The second case corresponds to a highly relativistic particle, i.e., $(\gamma\beta)^3 \gg \sigma\eta_0 R$, which, using the previous notation, means $\alpha \gg 1$. In this regime the main contribution to the integral is from the small values of x, a fact which justifies the expansion of the modified Bessel functions in Taylor series. Defining $y^2 \equiv \alpha^2 x^3/2$ we have

$$\mathcal{E}\simeq\frac{4\sqrt{2}}{3\pi}\int_0^\infty dy\,\frac{1}{1+y^2-y\sqrt{2}}$$
$$\simeq 2. \tag{28}$$

Therefore

$$\mathcal{E}\simeq\begin{cases}0.764\alpha & \text{for } \alpha\ll 1\\ 2 & \text{for } \alpha\gg 1\end{cases}. \tag{29}$$

As in the Čerenkov case, it indicates that for ultrarelativistic particles the decelerating force is independent of γ and of the material. However, the threshold for γ in this regime is much higher. In both cases the particle is decelerated by a force which corresponds to a (positive) image charge located at a distance $R/\sqrt{2}$ behind the electron.

In the low energy regime the decelerating force increases rapidly with the momentum of the particle. The square root of the conductivity indicates that this is a ''skin-depth'' effect. A clearer interpretation of this statement is obtained if we define the characteristic frequency $\omega_0\equiv 2c/R(\gamma\beta)^3$ by whose means

$$\delta=\sqrt{\frac{2}{\sigma\mu_0\omega_0}}, \tag{30}$$

and

$$\mathcal{E}\simeq\begin{cases}0.54\,\delta/R & \text{for } \delta\ll R\\ 2 & \text{for } \delta\gg R.\end{cases} \tag{31}$$

In the framework of this notation we observe that the characteristic frequency is low for very relativistic electrons and if the the skin depth is much larger than the radius then all the bulk material ''participates'' in the deceleration process. On the other hand, if the frequency is high, then the skin depth is small (comparing to the radius) and only a thin layer dissipates power, therefore the loss is proportional to δ. Finally, if the conductivity of the material is negative, this is to say that we have an active medium, then $\psi=5\pi/4$ and the force is accelerating, which means that energy can be transferred from the medium to the electron. This topic will be discussed next.

V. FORCE IN ACTIVE MEDIUM

The interaction of a moving ''macroparticle'' with a stationary two-state quantum system which consists of either atoms or molecules is considered here within the framework of the macroscopic (and scalar) dielectric coefficient. This coefficient is given by

$$\varepsilon_r(\omega)=1-\chi\,\frac{(\omega-\omega_0)T_2+j}{1+\xi^2+(\omega-\omega_0)^2T_2^2} \tag{32}$$

and it is tacitly assumed that the transients at the microscopic level are negligible. The macroparticle moves along a vacuum channel ''bored'' in an otherwise infinite dielectric medium. $\omega_0/2\pi$ is the resonant frequency of the medium, T_2 is the spectral linewidth, $\chi=\mu^2\Delta N T_2/\varepsilon_0\hbar$ is the normalized population inversion (and it is negative in this case); μ is the atom's dipole moment, $\Delta N\equiv N_1-N_2$ is the density of the population difference—subscript 1 represents the lower energy state and subscript 2 the higher one. Changes in the population difference due to energy transfer are considered here through the saturation term $\xi^2=(E/E_{sat})^2$; E is the amplitude of the acting electric field and the saturation field is given by $E_{sat}=\hbar/\mu\sqrt{\tau T_2}$ where τ is the characteristic time in which the population reaches its equilibrium state. Note that the background dielectric coefficient is assumed to be unity, excluding in this way the possibility of generation of Čerenkov radiation.

For the analytic evaluation of the integral it is convenient to further simplify the model which describes the medium. Examining the dielectric coefficient in (32) we observe that its real part is unity at resonance and off resonance [$\mathrm{Re}(\varepsilon_r-1)$], is antisymmetric relative to resonance, and vanishes far away from this point. Consequently, we approximate the dielectric coefficient with one whose real part is unity at all frequencies and its imaginary part is constant in a window of frequencies around resonance. The width of this window is determined by the linewidth and is determined such that the area of this window is identical with that calculated from (32). Explicitly $\varepsilon_r(\omega)\simeq 1-j\overline{\sigma(\omega)}$ and

$$\overline{\sigma(\omega)}\equiv\overline{\sigma_0}\times\begin{cases}1 & \text{for } |\omega-\omega_0|T_2<\pi/2\\ 0 & \text{for } |\omega-\omega_0|T_2>\pi/2\end{cases}, \tag{33}$$

where $\overline{\sigma_0}=\chi/(1+\zeta^2)$.

With this definition the normalized impedance reads

$$\zeta(x)=\frac{1}{1-j\overline{\sigma(x)}}\sqrt{1+j(\gamma\beta)^2\overline{\sigma(x)}}$$
$$\times\frac{K_0[x\sqrt{1+j(\gamma\beta)^2\overline{\sigma(x)}}]}{K_1[x\sqrt{1+j(\gamma\beta)^2\overline{\sigma(x)}}]}. \tag{34}$$

We consider the relativistic case such that $(\gamma\beta)^2|\overline{\sigma_0}|\gg 1$; moreover, at the typical frequencies of interest we assume that $\omega_0R|\overline{\sigma_0}|^{1/2}/c\ll 1$. Consequently, the normalized impedance function is

$$\zeta(x)\simeq jx\gamma^2\overline{\sigma(x)}. \tag{35}$$

This implies that $|\zeta|=x\gamma^2|\overline{\sigma}|$ and the phase of ζ is given by

$$\psi=\begin{cases}\pi/2 & \text{for } \overline{\sigma_0}>0\\ 0 & \text{for } \overline{\sigma_0}=0\\ -\pi/2 & \text{for } \overline{\sigma_0}<0,\end{cases} \tag{36}$$

where we assumed that $|\overline{\sigma}|<1$, thus $\psi=(\pi/2)\mathrm{sgn}(\overline{\sigma})$. The normalized gradient according to (16) is

$$\mathcal{E}=\frac{2}{\pi}\int_0^\infty dx\,\frac{x\gamma^2\overline{\sigma}(x)}{I_0^2(x)+x^2\gamma^4\overline{\sigma}^2(x)I_1^2(x)}. \tag{37}$$

From this last expression we observe that the contribution of the large x is small because of the exponential decay associated with the modified Bessel functions. Furthermore, if we assume that the particle is sufficiently relativistic and bearing in mind that in practice the integrand is nonzero only in a frequency domain determined by (33) then we can determine that if $\omega_0 R/c \ll \gamma$ then we can approximate $I_0(x) \simeq 1$ and $I_1(x) \simeq x/2$, thus

$$\mathscr{E} = \frac{2}{\pi} \int_{x_-}^{x_+} \frac{dx\ x\gamma^2 \overline{\sigma_0}}{1 + (x^2 \gamma^2 \overline{\sigma_0}/2)^2}, \tag{38}$$

where $x_\pm = R(\omega_0 \pm \pi/2T_2)/c\beta\gamma$. The integral can be evaluated analytically and the result is

$$\mathscr{E} = \frac{2}{\pi}\left[\arctan\left(\frac{1}{2}\ \overline{\sigma_0}\gamma^2 x_+^2\right) - \arctan\left(\frac{1}{2}\ \overline{\sigma_0}\gamma^2 x_-^2\right)\right]. \tag{39}$$

Substituting the explicit expression for $x_\pm$ and assuming that $\overline{\sigma}(\omega_0 R/c)^2 \ll 1$, we can approximate the inverse trigonometric function with its argument, thus

$$\mathscr{E} \simeq \frac{2}{\pi}\left(\frac{1}{2}\ \gamma^2 \overline{\sigma_0}\right)\left(\frac{R}{c\beta\gamma}\right)^2 \left[\left(\omega_0 + \frac{\pi}{2T_2}\right)^2 - \left(\omega_0 - \frac{\pi}{2T_2}\right)^2\right]$$

$$\simeq 2\overline{\sigma_0}\left(\frac{\omega_0}{c}\ R\right)\ \frac{R}{cT_2}. \tag{40}$$

This expression illustrates in an analytic form the fact that if the population is inverted then the gradient is negative corresponding to an accelerating force. If the population is not inverted the force slows down the electrons.

For completeness let us now consider the other extreme where $\omega_0 R|\sigma_0|^{1/2}/c \gg 1$ but still for a relativistic particle, thus $(\gamma\beta)^2|\sigma_0| \gg 1$. The normalized impedance function is

$$\zeta(x) \simeq \sqrt{j\gamma^2 \overline{\sigma(x)}}. \tag{41}$$

This implies that $|\zeta| = \gamma|\overline{\sigma}|^{1/2}$ and the phase of ζ is given by

$$\psi = \begin{cases} \pi/4 & \text{for } \overline{\sigma_0} > 0 \\ 0 & \text{for } \overline{\sigma_0} = 0 \\ -\pi/4 & \text{for } \overline{\sigma_0} < 0, \end{cases} \tag{42}$$

where we assumed that $|\overline{\sigma}| \ll 1$, thus $\psi = (\pi/4)\mathrm{sgn}(\overline{\sigma})$. The normalized gradient according to (16) is

$$\mathscr{E} = \frac{2}{\pi}\ \mathrm{sgn}(\overline{\sigma_0}) \int_{x_-}^{x_+} dx\ \frac{\gamma\sqrt{|\widetilde{\sigma}_0|/2}}{[I_0(x) + I_1(x)\gamma\sqrt{|\widetilde{\sigma}_0|/2}]^2 + [I_1(x)\gamma\sqrt{|\overline{\sigma_0}|/2}]^2}. \tag{43}$$

As in the previous cases, for highly relativistic particles, i.e., $\omega_0 R/c \ll \gamma$, the main contribution to the integral comes from the small values of x, therefore we use the asymptotic expression for the modified Bessel functions $[I_0(x) \simeq 1$ and $I_1(x) \simeq x/2]$ to write

$$\mathscr{E} = \frac{2}{\pi}\ \mathrm{sgn}(\overline{\sigma_0}) \int_{x_-}^{x_+} dx\ \frac{\gamma\sqrt{|\overline{\sigma_0}|/2}}{1 + \gamma^2|\overline{\sigma_0}|x^2/4 + x\gamma\sqrt{|\overline{\sigma_0}|/2}}. \tag{44}$$

The integral can be evaluated analytically and the result is

$$\mathscr{E} \simeq \frac{4}{\pi}\ \mathrm{sgn}(\overline{\sigma_0})\left\{\arctan\left[\sqrt{\frac{|\overline{\sigma_0}|}{2}}\frac{R}{c}\left(\omega_0 + \frac{\pi}{2T_2}\right)\right]\right.$$

$$\left. - \arctan\left[\sqrt{\frac{|\overline{\sigma_0}|}{2}}\frac{R}{c}\left(\omega_0 - \frac{\pi}{2T_2}\right)\right]\right\}. \tag{45}$$

Assuming that the frequency linewidth is relatively narrow $(\omega_0 T_2 \gg 1)$ and also that $\overline{\sigma_0}|(\omega_0 R/c)^2 \gg 1$, we can approximate (45) with

$$\mathscr{E} \simeq 4\left(\frac{R}{cT_2}\right)\left(\frac{\omega_0}{c}\ R\right)^{-2}\ \frac{\mathrm{sgn}(\overline{\sigma})}{\sqrt{|\overline{\sigma_0}|/2}}. \tag{46}$$

It is now instructive to summarize these two results [(40), (46)] from the perspective of one framework. For this purpose we use the definition of the skin depth in (30) with the conductivity defined as $\sigma \equiv |\overline{\sigma}|\omega_0\varepsilon_0$, thus $\delta = \sqrt{2/|\overline{\sigma_0}|(\omega_0/c)^2}$. With this definition we get

$$\mathscr{E} \simeq \mathrm{sgn}(\overline{\sigma})\ \frac{1}{\omega_0 T_2}\begin{cases} 4\delta/R & \text{for } \delta \ll R \\ 4(R/\delta)^2 & \text{for } \delta \gg R. \end{cases} \tag{47}$$

This expression is smaller than 2 (the asymptotic Čerenkov value) for most cases of interest, thus we should aim to avoid competition with the Čerenkov process since the latter is expected to be stronger.

Our efforts so far aimed to develop analytic expression for the reaction force of a medium as a relativistic electron traverses it. The effect, however, is not limited to relativistic electrons but it may occur also in the case of nonrelativistic particles. For this purpose let us consider (34) at the limit $(\gamma\beta)^2|\overline{\sigma_0}| \ll 1$, in which case the normalized impedance is given by

$$\zeta(x) = \frac{1}{1 - j\overline{\sigma(x)}}\ \frac{K_0(x)}{K_1(x)}, \tag{48}$$

if we further assume that $|\overline{\sigma_0}| \ll 1$ then the normalized gradient is given by

$$\mathscr{E} \simeq \sigma_0\ \frac{2}{\pi} \int_{x_-}^{x_+} dx\ x^2 K_0(x) K_1(x). \tag{49}$$

The integrand has a peak value for $x \simeq 0.4$, therefore in order to achieve maximum gradient it will be necessary that the integration domain (x_-, x_+) will be around this value. In other words,

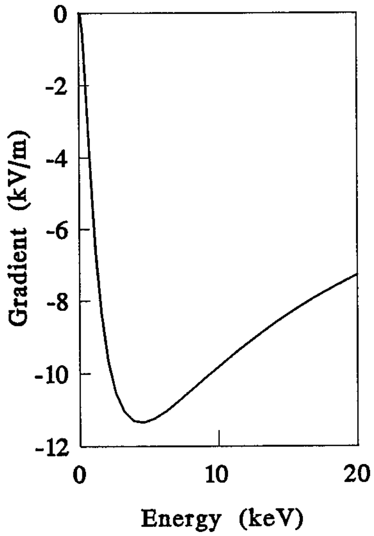

FIG. 4. Accelerating gradient at low energies. There is an optimal initial energy for which the gradient has a maximum value.

$$\frac{\omega_0}{c} R \sim 0.4\beta. \tag{50}$$

In Fig. 4 we illustrate the gradient which acts on a 10° bunch of electrons moving in a 0.2 mm channel bored in an active medium whose resonance is at $\lambda_0=10$ mm and the linewidth is determined by $\omega_0 T_2 \simeq 100$; we also assume that $n=1$, $\chi=-0.1$, and $\xi=0.1$. Note that for these parameters a bunch which has an initial energy of 4.5 keV experiences a peak acceleration gradient of about 11 kV/m. If we now assume that the refraction coefficient is not unity but $n=2$ then the shape of the curve is similar but the peak value occurs at an initial energy of 3.1 keV and the gradient is one order of magnitude smaller, namely, 1 kV/m. Thus even in the absence of Čerenkov radiation the background refraction coefficient has a decremental effect. The maximum gradient in this case is achieved when the linewidth is very large, i.e., $x_- \sim 0$ and $x_+ \sim \infty$ (corresponding to $\omega_0 T_2 \sim 1$). The normalized gradient in this case is given by

$$\mathscr{E} \simeq \frac{\chi}{\pi}. \tag{51}$$

Let us now examine the gradient at higher energies but also with much more energy stored in the medium. In other words, let us consider a medium whose $\lambda_0=1$ μm, $\omega_0 T_2 \sim 100$, and $n=1$. For effective interaction the radius of the channel has to be on the order of the radiation wavelength: in this case we chose $R=1$ μm. The other parameters are $N=10^8$, $\overline{\sigma_0}=-0.1$, and $n=1$. Figure 5 illustrates the gradient which acts on the particle as a function of its initial energy. For energies on the order of a few tens of MeV the gradient a bunch of 10^8 particles experiences is on the order of GV/m. One can therefore envision a system in which a bunch of electrons is injected into a chamber filled with active gas. At these energies the mean free path, even at high pressure, is much longer than the practical length of a typical acceleration module (1–10 m). In spite of these promising gradients there still remains the problem of competition from the Čerenkov effect.

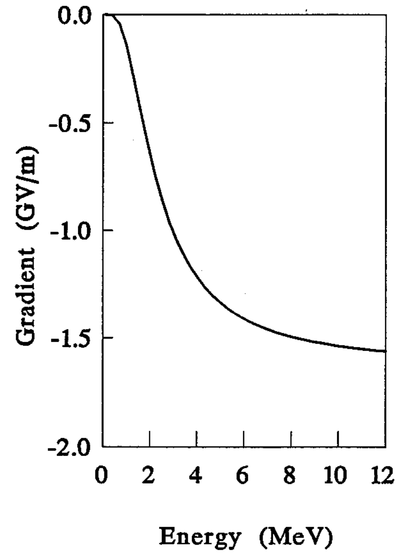

FIG. 5. Accelerating gradient at high energies. The real part of the refraction coefficient is assumed to be one.

In order to examine the feasibility of the method in a realistic medium we shall consider a gas whose molecules are assumed to have three resonances (four-level system) and only in one or at the most two, the population is inverted. The remainder serve as a model for the background refraction coefficient of the medium. Explicitly, the dielectric coefficient of the medium is assumed to have the form

$$\begin{aligned} \varepsilon_r(\omega) &= 1 - \sum_{i=0}^{2} \chi_i \frac{(\omega-\omega_{0,i})T_{2,i}+j}{1+\xi^2+(\omega-\omega_{0,i})^2 T_{2,i}^2} \\ &\simeq 1 - \sum_{i=0}^{2} \overline{\sigma_i}(\omega), \end{aligned} \tag{52}$$

where, similarly to (33),

$$\overline{\sigma_i}(\omega) \equiv \overline{\sigma_{0,i}} \times \begin{cases} 1 & \text{for } |\omega-\omega_{0,i}|T_{2,i} < \pi/2 \\ 0 & \text{for } |\omega-\omega_{0,i}|T_{2,i} > \pi/2 \end{cases} \tag{53}$$

and $\overline{\sigma_{0,i}}=\chi_i/(1+\xi^2)$. The parameters are as in the previous case, with the exception of the following, the three resonances are chosen at $\lambda_0=0.8$ μm, $\lambda_1=1.0$ μm, and $\lambda_2=1.3$ μm. The linewidth in each case satisfies $\omega_i T_{2,i}=100$. From the cases we examined we chose to present here six:

$$\begin{array}{llll} \text{(i)} & \chi_0=0.00, & \chi_1=-0.10, & \chi_2=0.00, \\ \text{(ii)} & \chi_0=0.00, & \chi_1=-0.10, & \chi_2=-0.05, \\ \text{(iii)} & \chi_0=0.05, & \chi_1=-0.10, & \chi_2=0.00, \\ \text{(iv)} & \chi_0=0.05, & \chi_1=-0.10, & \chi_2=0.10, \\ \text{(v)} & \chi_0=0.06, & \chi_1=-0.10, & \chi_2=0.01, \\ \text{(vi)} & \chi_0=-0.10, & \chi_1=-0.02, & \chi_2=0.10. \end{array} \tag{54}$$

These six cases are illustrated in Fig. 6. For reference to the previous discussion case (i) is identical with Fig. 5. The other two curves in the left frame illustrate the deceleration

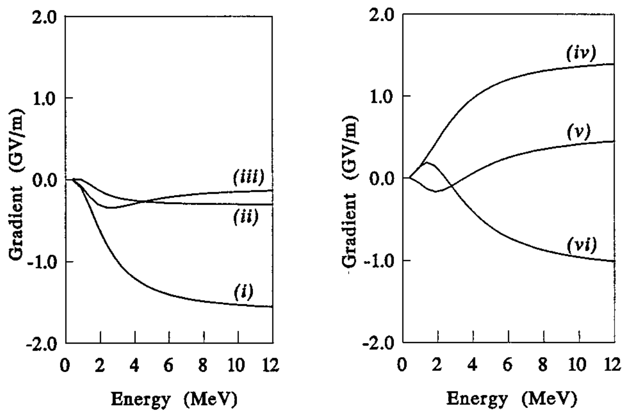

FIG. 6. Accelerating gradient in a system where three transitions are possible. The normalized population inversions corresponding to each case are presented in (54).

process associated with two resonances whose population is not inverted. The effect of shorter wavelength is, as expected, more pronounced and in this case the acceleration at high energies is very small. In the right frame we observe that if the population inversion is not sufficiently high comparing to the other two transitions then the bunch is actually decelerated—see (iv). Even if the population inversion of one transition is larger than the other two, acceleration is not guaranteed at high energies as revealed by (v). However, proper excitation of the material may ensure acceleration at high energies although at low energies the Čerenkov process is dominant—as revealed by (vi).

VI. DISCUSSION

In this study we examined the force which acts on a small bunch of electrons as it moves along a vacuum channel bored in a medium. It was shown that the reaction of the medium to the presence of the bunch depends strongly on the characteristics of the medium. In the dielectric case the medium is inert if the velocity of the electron is below the Čerenkov condition. As the particle exceeds this limit it is decelerated. When the medium is a lossy material, the bunch is slowed down from virtually zero velocity. The important result of this study is the proof that when the medium is *active*, the reaction field switches its phase and it accelerates the moving bunch. In other words, the bunch may receive energy initially stored in the medium. The process occurs at low as well as at high energies. In the latter case probably only the gas medium is relevant to the acceleration process due to the existence of a substantial refraction coefficient, in which case the motion is dominated by Čerenkov deceleration.

At low energies the electrons can also be accelerated and according to Fig. 4 there is clearly an optimal input energy for a given geometry and a given medium. In contrast to the highly relativistic case where high frequencies are preferable, the nonrelativistic regime operates with relatively low frequency systems (e.g., masers). The effect of the frequency on the gradient is set by two opposite trends: on one hand, the higher the frequency, the larger the energy stored in the medium and thus the larger the gradient. On the other hand, the gradient of an evanescent wave decays exponentially from the particle's location to the wall of the vacuum channel.

All the examples presented above assumed that $\omega T_2 \sim 100$, which is typically orders of magnitude smaller than actually used in (cw) lasers. Consequently the medium used for coherent generation of radiation is not necessarily adequate for acceleration. In fact, we should realize that there is a trade-off between choosing the conditions for the shortest T_2 possible in order to increase the gradient and a long T_2 which will allow enough time between the excitation of the medium and the passage of the bunch. This condition may become less stringent if a train of bunches is injected at the resonance of the medium.

Finally, as in the cases of passive dielectric or lossy medium at high energies the gradient is independent of the kinetic energy of the particles.

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